CLAIM:

Modus Ponens is probabilistically valid. That means that the uncertainty of the conclusion is less than or equal to the sum of the uncertainty of the premises.

Modus Ponens is this:

If A then B, A, so therefore B. Without begging any questions, let's symbolize this as
$A \rightarrow B, A \vdash B$

If we understand the conditional as the material conditional, then exactly the same arguments are probabilistically valid as are deductively valid. So for sure $A \supset B, A \vdash B$ is probabilistically valid. But let's check to make sure.

Label the four rows of the truth table a1, a2, a3, a4.

$P(A \supset B) = a1 + a3 + a4$ so uncertainty = $1-(a1 + a3 + a4) = a2$
$P(A) = a1 + a2$ so uncertainty = $1 - (a1 + a2) = a3 + a4$
$P(B) = a1 + a3$ so uncertainty = $1 - (a1 + a3) = a2 + a4$

Now the sum of the uncertainty of the premises is a2+a3+a4. Since each of these is ≥ 0, $a2+a3+a4 \geq a2 + a4$ which is the uncertainty of the conclusion.

Now if we understand the indicative conditional is some other way, it becomes unclear what probabilistic validity amounts to. One way to do this is to ask what would happen if instead of $P(A \rightarrow B)$ whatever that is, we simply replace this with $P(B|A)$ which is by definition $P(A \wedge B)/P(A)$

Then it is **ALSO** true that Modus Ponens will be probabilistically valid.

Proof:

$P(B|A) = P(A \wedge B)/P(A) = a1/(a1+a2)$ so the uncertainty is $1- [a1/(a1+a2)]$
$P(A) = a1 + a2$ so uncertainty = $1 - (a1 + a2) = a3 + a4$
$P(B) = a1 + a3$ so uncertainty = $1 - (a1 + a3) = a2 + a4$

I will now prove that $1- [a1/(a1+a2)] + a3 + a4 \geq a2 + a4$

$1- [a1/(a1+a2)] + a3 + a4 \geq a2 + a4$ **if and only if**
$1- [a1/(a1+a2)] + a3 \geq a2$ **[I subtracted a4 from both sides] - if and only if**
$1 + a3 - a2 \geq a1/(a1+a2)$ **[I added and subtracted from both sides] - if and only if**
$a1 + 2x\ a3 + a4 \geq a1/(a1+a2)$ **[replaced 1-a2] - if and only if**
$(a1+a2) (a1 + 2x\ a3 + a4) \geq a1$**- if and only if**
$a1(a1 + 2x\ a3 + a4) + a2 (a1 + 2x\ a3 + a4) \geq a1$

But this is definitely true. Notice that the left side is a1 times something and that thing is the sum of non-negative numbers. Then you add some more stuff. Again, all non-negative. So this is definitely true which means that the sum of the uncertainties in the premises is at least as great as the uncertainty of the conclusion. So Modus Ponens is probabilistically valid. A good inference. Reasons in favor of the premises are reasons for the conclusion, etc.